**N-body Simulations of Star-Star Encounters**

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**Abstract**

N-body simulations were carried out on a self-gravitating system, like the Solar System. A summary of all the sections is mentioned in the introduction. The basic N-body problem is described with all the equations used in the code. The motivation for writing the second order is explained in the application of a N-body code to various astrophysics problems. The basic second-order code is described later with all the initial conditions given to the system. The initial results obtained from successful runs of the code are also included. For the second semester, the project plan is added later. All the tasks to be done are represented in a gannt chart. Finally, a brief summary of the report is included.

**1. Introduction**

The basic N-body problem is described for a system where only gravitational interactions take place. These interactions cause the positions and velocities of the particles to evolve with time. On considering a stellar cluster where Newtonian gravity dominates between the stars, the evolution of a group of stars can be tracked. The aim of this project lies in developing a working N-body code which tracks the evolution of a star formation region.

For the initial part of the project, a simple second order code is created for the solar system. The various sections of the code is explained later. Certain tests are carried out on the code which give an idea of the error on the values of the dynamical properties of the bodies. Initial results from the simulations are produced, with different simulation times selected.

There are few astrophysics problems where N-body simulations are used. Certain problems like the mass segregation of a star cluster, orbital migration of the giant planets in the planetesimal disk, formation of the magellanic stream from the galactic cannibalism of the Magellanic clouds etcetera. The motivation for doing the N-body simulations come from these applications.

**2. Literature review**

**2.1 N-body problem**

Newton’s law of gravitation describes how a group of stars interact in a star cluster. These gravitational interactions cause the dynamical properties (velocity, position, acceleration) of the stars to change. This causes a change in the dynamical properties of the whole region. The evolution of this stellar region can be observed in a N-BODY simulation. The N-BODY problem is incorporates the prediction of future dynamical properties of the stars in the system. The initial properties of the particles in the system are known. In a system of N particles, the acceleration of a particle can be defined as,

-1

Where, mass of the test particle

the unit vector along the direction of the distance vector

the modulus square of the distance between the bodies considered

Integrating equation (1) provides the position and velocity of a particle at any time t. For *N*=2 the above equation is analytically solvable. Since we are considering stellar clusters as our system, where N2, numerical methods are considered.

Numerical integration of equation 1 provides the below solutions.

-2

-3

Where, and the new position and velocity of the particles.

and the initial position and velocity of the particles

, , , the initial acceleration of the particles with the latter three being the 1st, 2nd, 3rd time derivatives.

timestep for the simulation

The timestep *dt* determines the accuracy of the values of the future position and velocity of the particles. It goes inversely with the computational time. There is a rise in the number of calculations done in a single simulation as *dt* drops.

Accuracy in predicting the motion of stars in the cluster is the main task. The error in the solution is proportional to the timestep. As we go to higher orders, a small drop in *dt* will imply a large reduction in the error.

For our project we consider the 2nd order method, with the 4th order predictor-corrector (Hermite scheme) method used in semester 2. Below are the equations used for the second order method.

-4

-5

The 4th order predictor-corrector method improves on the accuracy of the previous methods.

**2.2 Astrophysics applications of N-BODY method**

The motivation of writing this N-body code is due to its application to certain astrophysics problems. The problems are: mass segregation in star clusters, the orbital migration of the giant planets in the planetesimal disk, presence of boxy and peanut-shaped bars in the bar evolution phase. In these problems, we have the presence of self-gravitating system. Newtonian gravity is the dominant factor in the system.

Star cluster evolution is a difficult scenario to build in a numerical simulation. The interaction between the stars determine the evolution of the cluster. In Khalisi, E., Amaro-Seoane, P. and Spurzem, R. (2006) the dynamical evolution of the star cluster was carried out by considering two different stellar mass groups. In dynamical equilibrium, the evolution of the cluster depends on it achieving a thermal velocity distribution through small changes in the velocity of the stars. This phenomenon is termed as relaxation. Relaxation forms the major part in shaping the structure of the cluster. The important phase in the evolution of the cluster comes during the central core collapse. Three processes are able to reach this stage: equipartition, evaporation and gravothermal instability. Star-star encounters allow mass segregation in the central regions. This leads to movement in the heavy and low mass components to and away from the centre of the star cluster. This movement is achieved through transfer of Kinetic energy from the heavy to light component. There is a thermal energy outflow from the center to the outer regions. Significant evidence for mass segregation in star clusters was found in Infra-red observations of the trapezium cluster in Orion.

A simplest approximation of two-mass component simulations is considered. To carry out the simulations, they considered a plummer sphere model of the density distribution in the cluster. with the particles considered point-like, close encounters are not avoided. In addition, other stellar evolution scenarios, like primordial binaries, tidal fields etc. They use two parameters, the fraction of heavy mass component, and the mass fraction of each particle, to describe each model. A number of runs were assigned to each model, with each of them having a different setup of initial positions and velocities for all the particles. A strong approach was done to produce accurate N-body simulations with a higher number of particles, keeping in mind about a low computing power. With mass segregation in stellar clusters used for N-body simulations, the orbital migration of the giant planets in a planetesimal disk also forms a N-body problem.

The Nice model explains the orbital migration of the giant planets and the current structure of the outer Solar System. Specifically the orbital migration of Neptune caused the for majority of KBOs at Neptune’s mean motion resonance of 5:2. Also the large eccentricity in the Pluto’s orbit came from this migration. The Kuiper belt inhabits a vast number of residual planetesimals which are at various resonances. These Kuiper Belt Objects form a relic of the history of the outer solar system. The occurrence of the KBOs came from the orbital migration of the giant planets. The interaction between the planetesimal disk and the planets led to exchange of angular momentum thereby affecting the orbits.

N-body simulations of the interactions between the giant planets and the planetesimal disk was carried out (Hahn, J.M. and Malhotra, R., 2005, p. 2392). The simulations included massless particles surrounding the migrating giant planets. With a timescale of the age of the solar system, the orbital evolution was tracked. Two scenarios for the initial disk were considered: when it was dynamically cold, and the final where it was destabilized before migration. A MERCURY6 N-body integrator for the particular evolution process. The migration was demonstrated by applying an external torque to the semi-major axis of the orbit. Below is the time-dependent form of the semi-major axis,

Where, the final semi-major axis of the planet

planet’s net radial displacement

e-fold timescale for planet migration

For the initial conditions, the current masses and orbits of the planets was considered. In addition to that, the initial semi-major axis was modified by applying . This allowed for the torque to return the orbits to their present configurations. Also, various constraints were used for for each planet’s orbit depending on the different resonances. A timestep of 0.5 years was used, with the massless particles randomly distributed over a range of 20-80 AU. A more extensive approach for the N-body simulations was done by Hahn, J.M. and Malhotra, R. (1999, p. 3041) to look at the orbital migration of the giant planets in the remnant planetesimal disk. Moreover, different cases for the migration were considered based on the mass of the planetesimal disk.

Another case which can be explained using N-body simulations is stage after the runaway growth. The core accretion theory for planet formation gives a good description of the runaway growth and the oligarchic stages. With the protoplanets created, their orbital evolution among the remaining planetesimals was explored using 3D N-body simulations by Kokubo, E. and Ida, S. (1998, p.171). They observed the growth of the protoplanets slower among them, whereas when compared to the growth of the planetesimals. A formation of planet-planetesimal system was explored.

The formation of the magellanic stream near the Milky Way through galactic cannibalism is produced using numerical simulations (Maddison, S.T., Kawata, D. and Gibson, B.K., 2002, pp.421-422). The tidal gravitational field formed from the merger of the MW and the Large and Small Magellanic Clouds were the cause for this stream. They compared two types of numerical simulations, them being the N-body only merger and hydrodynamic one. For the latter type, the factors of star formation, supernova feedback and metal enrichment were included into the simulations along with the gravitational interactions. The results from the N-body only simulations showed the SMC being tidally stripped which gave the presence of the magellanic stream. However, from the second simulation the SMC was strongly disrupted with the stream devoid of stars. Figure 1 explains these results. The observations of the magellanic stream depict most of the gas in the stream.

A close up of a map

Description automatically generated

Figure 1: the different panels show the results from the N-body only and hydrodynamical simulations of the three-galaxy merger. Left two panels- N-body-only results showing the magellanic stream. Three right panels- hydrodynamical simulations results show the presence of gas in the magellanic stream.

**3. Progress on project (LOOK AT THE REPORT DOC. ON THE UNIVERSITY PC. CHECK THE PROGRESS SECTION)**

The initial work on the project was carried by constructing a simple second order code. Equation 5 can be corrected by considering an assumption.

-6

Initially a three-body system of Sun-Earth-Jupiter was considered for the problem, with the rest of the planets added later.

For the first block of the code, the declaration of all the variables used was done. For certain known variables like the gravitational constant *G*, the mass, velocity and initial positions of the planets, the initialisation is done. From the planetary fact sheet on the NASA website[1], the initial positions and velocities were taken. Certain initial conditions for the position and velocity are mentioned below. From these conditions, the orbits of the planets were forced to be on the XY plane.

For this system, we added the code which did centre-of-mass and velocity corrections. Without including this correction, the system would drift away from the origin during the simulations. The centre-of-mass and centre-of-velocity in the x direction is calculated as below,

-7

-8

Where, , is the total mass of the system

An infinite loop was created which would run the simulations to a time , with a time-step *dt.* Future positions and velocities determined using equation (4) and (6). Modified form of equation (1) was used to calculate the future accelerations.

-9

Where,

The orbits of the planets were produced where the runtime for the simulations was 1000 yr. The timestep was chosen to be 1000 sec. Due to the large runtime of 1000 yrs, the compilation took around 4-5 minutes.

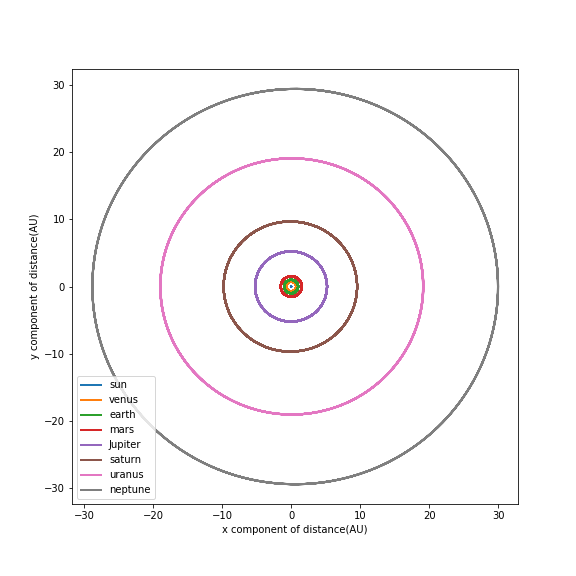


Figure 1: Orbits of the planets in the solar system.

To check the stability of this system, an energy conservation check was done. This was carried out by determining the fractional energy of this system using the equations below. A counter was set so that after every 6 months this calculation was possible.

-10

-11

-12

Where,

total energy of the system at time *t*

and the fractional energy and the initial energy of the system respectively

A plot of the fractional energy varying with time was produced for a time step, seconds.

A screenshot of a cell phone

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Figure 1: fractional energy of the system against time (in yrs). The simulation was run for 1000 yrs with a time-step of 1000 seconds.

The fractional energy of the system depends on the timestep, dt. For good accuracy, the time-step should be very low. This significantly increases the computation time, making the second order code too slow. For getting to Million yr timescales, the fourth order method presents a good method.

Another test for the code was done by looking at the distances from the sun varying with time for the planets. For every 1000th iteration, the distance was calculated for the bodies. This helped reduce the computation time.

A screenshot of a social media post

Description automatically generated Figure 2: The separation from the sun (in AU) against time (in yrs). The periodic variations present in the curves is due to the interactions between the planets.

**4. Project plan**

At the start of week 1, the core part of the work in semester 2 begins with task1, i.e., building a basic fourth order predictor-corrector code. A time of 2-3 weeks is given for this task based on its difficulty. After forming the base, we require an adaptive timestep for the code (task 2). Depending on the errors obtained from the energy checks, the code either doubles or halves the timestep, *dt*. A similar amount of time of 3 weeks is assigned to this task. The two main components of the code are ready. This leads to task 3 of testing the code. The results obtained from these tests determine whether the code is working. For example, we should observe milankovic cycles when we produce the plot of the orbits of the planets in our Solar System.

Task 4 provides the motivation of writing this piece of code. We apply this code to any astrophysics problem (like late oligarchic phase of planet formation). Moreover, certain tweaks are added at this point to increase the speed of the code. With a time of 2 weeks, it overlaps with the easter break. Then we lead to the most important task 5, the write-up of the report. All the figures and results produced in tasks 3 and 4 are included in the report. Tasks 4 and 5 overlap at the beginning of easter break. A time of 3-4 weeks is assigned so that a draft of the final report can be submitted to the supervisor approximately two weeks before the deadline (17/05/2019). The final task involves in refining the report. Within a week the report is checked for any mistakes before the final submission.

**5. Conclusion**

A simple second order code for the N-body simulations was constructed to apply on the current Solar System. The necessary initial conditions were applied with the code executed for two simulation times. Initial results were produced from certain tests (tests on the stability of the system and energy conservation) on the code.

**References**

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